A New Hybrid Model For Borefield Heat Exchangers Performance Evaluation

Picard Damien M.D. **Helsen Lieve Ph.D.**

ABSTRACT HEADING

Ground coupled heat pump systems are gaining importance for heating and cooling of buildings. To assess their performance, detailed simulation models are required with both short-term (to calculate the instantaneous coefficient of performance of the heat pump) and long-term (to calculate the ground temperature evolution) accuracy. A step response is calculated using a combination of a short-term response model which takes into account the transient heat transfer in the heat carrier fluid, the grout and the immediately surrounding ground, and a long-term response model which calculates the boreholes interactions. The state-of-the-art of both models has been chosen from an extensive literature review. Moreover, an aggregation method is implemented to speed up the calculations. Validation shows good results and very high computational efficiency.

INTRODUCTION

Energy system simulations for design or performance evaluation of buildings and communities have gained significant importance in the last decades resulting in several dynamic simulation platforms such as EnergyPlus [1], TRNSYS [2] or Modelica [11]. These simulation tools should comprise libraries containing every important building system element including for example accurate and computationally efficient borefield models.

TRNSYS already comprises different borefield models. The Superposition Borehole Model (SBM) [22], developed by Hellström, is a detailed three-dimensional model for the transient thermal process in a borefield. The model allows to simulate single or multiple, vertical or inclined boreholes. The dynamics of the borehole heat exchanger (BHX) (i.e. from the heat carrier fluid (HCF) to the borehole wall) is not taken into account and the computation time is very high. A simplification of SBM is the Duct Heat Storage model (DST), also developed by Hellström [13] which calculates the transient thermal process for multiple borehole configurations, uniformly positioned in a cylindrical volume. The model does not take the dynamics of the BHX into account but it is fast and it calculates the interaction between the boreholes (it uses pre-computed g-functions obtained by the SBM). Its TRNSYS implementation (type 557) [22] can be used together with a separate program called BORE to calculate the borehole thermal resistance depending on the flow rate and the temperature. Finally, TRNSYS comprises Finally, TRNSYS comprises the type 451a [14] based on a previous version of the EWS program [25]. This model can simulate a single vertical BHX with coaxial, single- or double-U-pipe design. The transient heat flux in the ground, in the filling material, and in the HCF are taken into account. However, it is unclear which capacities are used and where are they located. The earth is divided in several horizontal layers, each having its own thermal properties. The model also handles multiple borehole simulations using numerical and analytical *g-functions* and superposition. The analytical *g-functions* are calculated with Eskilson's line-source approach or with the Cylindrical Source model [14]. Both methods overestimate the long-term temperature response (see Section 2.2).

To the authors' knowledge, no borefield model has been implemented in Modelica© so far. The open-source Modelica Buildings library [23] developed by the Lawrens Berkeley National Laboratory (LBNL, US) is the only freely available library which has a single-U-pipe single borehole model. The borehole model is similar to the EWS model implemented in TRNSYS. The HCF and the grout (i.e. the filling material of the borehole) are simulated dynamically but their capacities are lumped. A triangle thermal resistance network is used to describe the heat transfer into the BHX. The model is not suited for multiple borehole simulation. The E.ON Energy Research Center (Germany) [15] also developed a single borehole model for single-U-pipe and coaxial type. The pipe model is connected to an axially and radially discretized cylindrical ground model. A fixed temperature boundary condition is used for the ground model. The model does not take

Picard Damien is Ph.D. Student at the Division of Applied Mechanics and Energy Conversion, Department of Mechanical Engineering, K.U.Leuven, Belgium. **Helsen Lieve** is a professor at the Department of Mechanical Engineering, K.U.Leuven, Belgium.

the dynamics of the grout into account and multiple borehole simulation is not possible.

To the author's knowledge, no model has been implemented in building simulation tools so far, which (i) is able to simulate any arbitrary configuration of boreholes, (ii) allows coaxial, single-U- or double-U-tube type BHX, (iii) has short- and long-term accuracy for minute-based year-long simulations, (iv), is numerically efficient and (v) is available in a general energy system simulation program to evaluate system performance. The aim of this paper is to propose a new borefield model, implemented in Modelica, which meets the above mentioned requirements. No ground water flow is assumed.

Section 2 gives an overview of the existing models in the literature and Section 3 describes the new implemented model. Finally, Section 4 and 5 validate the model and give an example including a CPU-time comparison with the existing borehole model of the Buildings library. The main conclusions are summarized in section 6.

EXISTING MODELS

In general, borefield models can be divided into two groups, i.e. the short-term models (STM) which focus on the transient heat transfer within the grout and the immediate ground (see Section 2.1) and the long-term models (LTM) which describe the transient heat transfer in the surrounding ground (see Section 2.2). The models of both groups can be classified as (1) analytical models, (2) numerical models using finite-volumes and (3) empirical models.

Short-term models

STM describe the transient heat flux in the BHX and the immediate surrounding ground. For the steady state case, Hellström [13] defined two thermal resistances to describe the heat flux, i.e. the *fluid-to-ground resistance* R_b, and the *grout-to-grout resistance* R_a. R_b is defined as the resistance from the HCF in the pipes (with each pipe having an equal HCF temperature) to the borehole wall with uniform equivalent temperature. A correction factor can be used to calculate the *effective fluid-to-ground thermal resistance* R^*_b which includes the effects of the varying HCF temperature along the flow channels (extrapolation from the 2D resistance to 3D) and the thermal interaction between them. R_a represents the thermal interaction between the different grout parts of the borehole. Hellström calculated both resistances using the complex *multipole* method developed by Bennet and Claesson [6]. Many authors proposed alternative and simplified methods to calculate these resistances, so did Hellström himself [13], using a line-source in composite region approximation, or Paul with his famous, experimentally determined shape factor coefficients which depend on the shank spacing [9]. Lamarche et al. [19] concluded in their excellent review paper that the multipole method of Claesson and Bennet gives the most accurate results except when the borehole is lined with a high conductivity steel casing, for which the method of Sharqawy et al. is a better choice. In the latter the borehole resistance is indeed decreased by the fin effect of the steel casing. However, none of the methods is accurate when the grout and ground conductivity are similar or when the legs of the U-pipes are very close to each other. Luckily these two scenarios are unusual. Finally they advise to use the correction factors proposed by Hellström and by Zeng et al. to calculate R^*_b (see [19]).

In order to describe the transient behaviour of the borehole, the dynamics of the HCF, of the pipe wall and mainly of the grout should be taken into account. Yavuzturk et al. [24] developed a numerical model based on a two-dimensional fully implicit finite volume formulation in polar coordinates to calculate the short-term thermal response of the borehole. The response is then used to extend the dimensionless *g-functions* (see Section 2.2) for short time scales. The method shows a relative temperature error of about 3% after one hour and about 0.15% after 24h due to the poor fit of the numerical grid to the actual borehole inner geometry. Lamarche [18] developed a fully 3D numerical model including the ground, grout, pipes and fluid using the COMSOL™ finite element software. A very fine adaptive grid is used and the model serves as test bench. Both approaches are very time-consuming due to the very high number of grid cells.

Several authors have developed analytical or empirical transient borehole models to speed up calculations. So far, analytical solutions exist only for boreholes with equivalent pipe diameter, i.e. the borehole is approximated as a single (lumped) pipe in the middle of the grout. Different formulations exist to compute the equivalent diameter as shown by Lamarche [18] or by Chiasson [9]. Gu and O'Neal extended the cylindrical heat source solution of Carslaw and Jaeger to allow multiple layers (see [9]). Young [9] proposed an analytical solution based on the buried cable solution with the core representing the fluid, the isolator the borehole resistance and the sheath the borehole wall. He adapted the solution in order to add the grout capacity to the core and the sheath to improve the approximation. Lamarche [18] found the exact solution of the borehole with equivalent diameter in the Laplace domain. The solution does not take the HCF into account. Finally, Javed and Claesson [16] developed an analytical solution including the HCF capacity. These analytical models approximate the behaviour of single- or double-U-type only and their accuracy depends on the grout and soil properties as well as on the equivalent diameter correlation. Bauer et al. [3] proposed a different method by setting up thermal resistive-capacitive models (TRCM) for coaxial, single- and double-U-tube. The resistances are calculated such that the R_b and R_a of the BHX correspond to their respective values from the multipole method of Claesson and Bennet and the sum of the capacities equals the total grout thermal capacity. A very good agreement between the numerical models and the TRCMs is observed after 15 minutes when the capacities are placed at the grout area centers (see Section 2.1).

Long-term models

The STMs described in Section 2.1 cannot accurately simulate the transient heat transfer into the ground and the interactions between the different boreholes. The STMs need the borehole wall temperature T_b as an input. LTMs are designed to calculate T_b accurately over periods longer than decades. Most models are based on the step response of the heat transfer rate and use the superposition principle to compute the response to an arbitrary load profile.

Long-term behaviour of multiple borehole systems has been investigated firstly by Eskilson and Hellström [12, 13]. Eskilson developed a two-dimensional finite difference model in radial-axial coordinates for vertical or inclined boreholes. The model allows to calculate the heat flux $q(t)$ through the borehole wall for a given uniform $T_b(t)$, or $T_b(t,z)$ for a given $q(t)$, with z the axial coordinate. This is an approximation of reality in the case of a non-coaxial borehole where T_b is not radially-uniform. The author used this model to compute the famous *g-functions* defined as $g(t/t_s, r_b/H) = (2\pi k(T_b - T_0))/q'_0$ with $t_s = H^2/(9a)$, H the depth of the borehole, a and k the heat diffusivity and the conductivity of the ground, r_b the diameter of the BHX, T_0 the initial uniform ground temperature, and q'_0 the heat flow rate per meter. The *g-functions* are valid for $t > (5 r_b^2)/a$. The response to an arbitrary load is then obtained by approximating the load as a sum of time-shifted step-loads and taking the sum of their responses (see Section 3.3).

Claesson and Eskilson [12] also described an analytical approximation of the *g-functions* using a finite-line source approach and the method of images to ensure zero heat flux at the ground's earth surface. This model uses a prescribed heat flux at the borehole wall instead of prescribed temperature, and it computes the $T_b(t)$ at $z=H/2$ as the representative borehole wall temperature. However, they observed that the analytical solution overestimates the temperature response compared to the numerical *g-functions*. Zeng et al. and Lamarche and Beauchamp (see [17]) could solve that problem by taking the average (in depth) T_b as representative temperature. Lamarche and Beauchamp made considerable effort to mathematically simplify their analytical solution in order to allow fast computing. Their solution is about 1000 times faster than the solution of Zeng et al. and about 10 times faster than the solution of Claesson and Eskilson. The analytical and numerical *g-functions* show very good agreement for the case of a single borehole. Claesson and Javed [10] developed in a similar way an analytical model to calculate the mean T_b. Their solution is even more compact than Lamarche and Beauchamp's expression (see Section 3). All models mentioned in this paragraph are extended to multiple borehole models using spatial superposition approximation. As mentioned by Eskilson and Claesson [22], the superposition does not respect the exact boundary condition. Indeed, the superposition introduces a non-zero heat flux which is not due to heat injection, at each borehole location. Furthermore, contrary to numerical models, analytical models assume that the heat injection/extraction is the same for each borehole. This is obviously not the case because the temperature field within the borefield is usually non-uniform and the convective-diffusive heat transfer from each borehole is driven by the temperature difference. The injected/extracted heat of a borehole in the middle of the configuration differs from the one at the edge. This is illustrated by Malayappan and Spitler [20], who showed that it can cause a serious deviation from the numerical *g-functions* for compact configurations. For example, if $k=2\ W/mK\ (13.86\ BTU.in/(hr.ft^2.F))$, $q'_0=50\ W/m$ (48 Btu/h.ft^2)and a square borefield configuration with $a \times b$ boreholes with $B/H=0.0625$ is used, the analytical solution of Claesson and Javed overestimates the temperature rise due to the step input with 0.25 K (for 4x4), 0.5 K (for 6x6) and 1.76 K (for 8x8) after 44 years [20]. The error for a variable load will be much lower.

NEW BOREFIELD

This paper proposes a new model that combines a STM with a LTM to obtain accurate g-functions. Section 3.1 and 3.2 describe the implementation of the STM and the LTM and Section 3.3 deals with their combination to a single hybrid model.

Short-term model

The short-term model (STM) should be able to calculate the transient thermal response of the HCF, the grout and the surrounding ground accurately for time periods ranging from minutes to $t = 5\ r_b^2\ /\ a$ (typically 10h < t < 200 h). The interaction between the boreholes for these short times can be neglected; therefore a single borehole model is used.

Bauer's RCM is an appropriate choice for the BHX of the STM [3]. Its steady state resistances (R_b and R_a) are indeed calculated with the most accurate method (i.e. the multipole method of Bennet and Claesson [6]), it includes the dynamics of the grout and the authors propose models for coaxial, single- and double-U-type system. The position of the capacities is calculated to be at the area center of the borehole with an equivalent single pipe. The dynamics of the HCF is calculated using the *Fluid* base classes of the open-source Buildings library [23] and the *Media* library from the Modelica Standard Library [11]. The convective resistance between the HCF and the pipe is calculated by the correlation for smooth pipes in turbulent flow regime of Dittus-Boelter in the case of single- and double-U-tubes [13]. For the circular-tube annulus, the correlation of Petuhkov and Roizen is used [13]. Vertical discretization is also possible but no vertical heat transfer is computed except through the HCF. Finally, the heat transfer from the borehole wall to the surrounding ground is calculated by discretizing the ground using a TRCM. The mesh is generated according to Eskilson's guidelines [12]:

$$\Delta r = \left[\Delta r_{min}, \Delta r_{min}, \Delta r_{min}, \beta\Delta r_{min}, \beta^2\Delta r_{min}, \cdots\right] \quad \text{with} \quad \Delta r_{min} = \min(\sqrt{\alpha\Delta t_{min}}, H/5)$$

with Δt_{min} the minimum resolution time and Δr the size of the cell. The discretization has been tested with the analytical Cylindrical Source Model developed by Carslaw and Jaeger [8] and it shows very good agreement when the mesh is chosen fine enough (i.e. around 10 states for a layer of 3 meters). The external part of the ground layer is connected to a constant undisturbed ground temperature. Fig. 1 illustrates the STM structure for a single borehole with a single-U-tube configuration.

Long-term response model

The long-term temperature response of the borefield is calculated using the model of Javed and Claesson [16]. Their model is preferred for its accuracy and efficiency and the analytical approach is chosen to enable arbitrary borefield configurations.

The model is based on the spatial superposition of finite line-sources of equal length, each representing one borehole of the borefield. The finite line-source is calculated from the convolution of a point source of constant power along the depth of the borefield. The mirror of the solution at z=0 is subtracted to ensure that no heat transfer occurs between the ground and the ambient air. After several mathematical manipulations to simplify the calculation, Javed and Claesson obtain the following compact expression for the mean borehole wall temperature:

$$\bar{T}_{mbhw}(t) = \frac{q_0}{4\pi k}\int_{1/\sqrt{4\alpha t}}^{\infty}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} e^{-r_{i,j}^2 s^2}\right)\frac{\mathrm{I_{ls}}(Hs)}{Hs^2}ds \tag{1}$$

where q_0 is the heat flux per meter length, k is the ground heat conductivity, a is the ground heat diffusivity ($k\ /\ (\rho\ c_p)$),

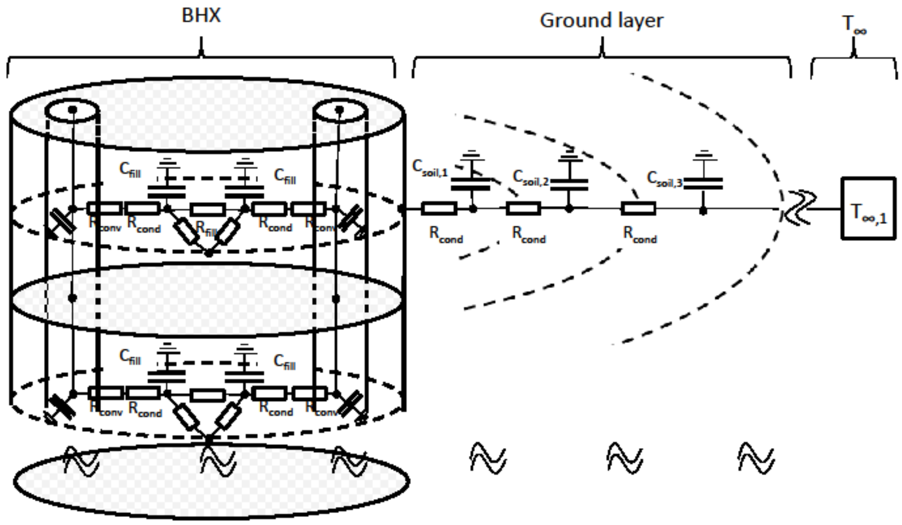

Figure 1: *Structure of the short-term model for a single borehole with a single-U-tube configuration.*

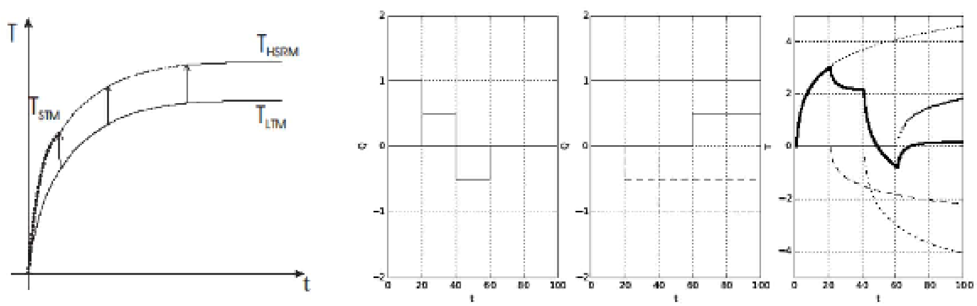

Figure 2: *(a) Combination of the long-term temperature step response T_{LTM} with the short-term temperature step response T_{STM} to compose the hybrid temperature step response T_{HSRM}). (b) Left: initial discrete load; center: decomposition of the load into a sum of time-shifted step loads; right: temperature response to the load obtained by superposition.*

N is the number of boreholes and H is the depth of the borefield (equal for all boreholes). I_{ls} is defined by Eq. 2 and $r_{i,j}$ by Eq. 3.

$$I_{ls}(h) = 4\ \mathrm{ierf}(h) - \mathrm{ierf}(2h) \quad \text{with} \quad \mathrm{ierf}(x) = \int_0^x \mathrm{erf}(u)du = x\ \mathrm{erf}(x) - \frac{1}{\sqrt{\pi}}(1 - e^{-x^2}) \tag{2}$$

where *erf* is the error function,

$$r_{i,j} = \begin{cases} r_b & \text{if } i = i \\ \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} & \text{if } i \neq j \end{cases} \tag{3}$$

where r_b is the BHX radius and (x_i, y_i) are the spatial coordinates of the center of each borehole from an arbitrary reference point.

Eq. 1 is valid for $t > 5\ r_b^2\ /\ a$, i.e after the transient part of the heat transfer through the BHX is completed [12]. The model also makes an important approximation by assuming uniform heat flux for all boreholes. The (long-term) accuracy of the model decreases for long simulation times for configurations with non-uniform heat fluxes, e.g. densely packed rectangular grid (see Section 2.2).

Computation of the response function and aggregation method

The STM and LTM are now combined to compute a step response with both short- (dynamics of BHX) and long-term (interaction between the boreholes) accuracy to give a new hybrid step-response model (HSRM) (see Fig. 2). The LTM response is lifted to the STM response in the interval where both models have the same behaviour, i.e. after the transient behaviour of the BHX and before the boreholes start to thermally interact. As Javed [16] mentioned in his work, this interval is relatively large.

As described above, g-functions and most of the analytical models give only a step response solution for the borefield. In order to model arbitrary input signals, the inputs need to be represented by a sum of time-shifted step signals and their responses should be superposed (see Fig. 3). For minute-based multi-year simulations where the individual step responses of each input step should be summed, this approach leads to expensive calculations. This problem is solved by using an aggregation method. A literature study of the different aggregation techniques is beyond the scope of this paper. We implemented the method proposed by Claesson and Javed which is described in [16] or in [21].

MODEL VALIDATION

The STM and the LTM have been verified by their respective developers. To avoid coding error and to check and generalize the validity of the model, an extensive model verification is carried out.

The STM is compared to the widely used sandbox experiment of Beier et al. [5]. These authors have carefully performed a thermal response test using a U-tube BHX. The U-tube is grouted into an aluminium pipe of 18 meters long which is placed into a box filled with homogeneous sand. An electrical heater injects a constant power to the HCF and a pump ensures a constant flow rate. All ground and grout properties are presented in their paper, except the heat capacities. The ground capacity has been estimated by Beier [4] using a best fit method (c_v = 3.2 MJ/m^{3}K). For the grout a heat capacity of 4 MJ/m^{3}K is used. The HCF temperature is measured at the in- and outlet as well as the BHX wall and sand temperatures at various depths. It should be noted that the aluminium pipe around the grout acts as a thermal fin which reduces the borehole thermal resistance by evening out its wall temperature. Consequently the HCF temperature should be lower for the experiment compared to the models which do not take this fin effect into account (see [19]). Fig. 3 compares the average of the in- and outlet temperatures of the HCF for the case of the experiment, the Buildings model, TRNSYS models (type 557a (DST)) and the implemented HSRM. The Buildings model dynamics is clearly to slow. This is due to the position of the lumped capacity of the grout, as illustrated by Bauer et al [3]. In the Buildings model, the grout capacities are positioned at the pipe wall instead of the area center of each grout zone. Adapting the capcity location (which requires also the adaptation of the resistances), the problem is solved (*Buildings adapted*). TRNSYS DST model and HSRM give similar results. DST, however, does not incorporate the short-term thermal dynamics of the fluid, contrary to the new model HSRM.

The LTM is verified using the well known g-function developed by Eskilson and the infinite cylindrical heat source (CHS) solution for different configurations (the data are taken from the paper of Bertagnolio et al. [7]). Fig. 4 illustrates the case of a 110 m deep (328 ft) single borehole. The error of the implemented model compared to the g-function never exceeds 0.11 K during the 25 year-long simulation (relative error of 1.7 %). The difference is caused by the so-called end effect of the borehole. The analytical solution uses a finite line-source approximation whereas the Eskilson finite volume model is three-dimensional (different boundary condition at the foot of each borehole). The CHS model is logically unable to model the end effect. Fig. 4 illustrates the case of a borefield with a square 8x8 configuration. The length of the boreholes is 110 m (328 ft) and the distance between the boreholes to length ratio equals 0.05. Due to the very compact configuration, high ground conductivity and low heat injection, a large error appears, as Malayappan and Spitler [20] warned

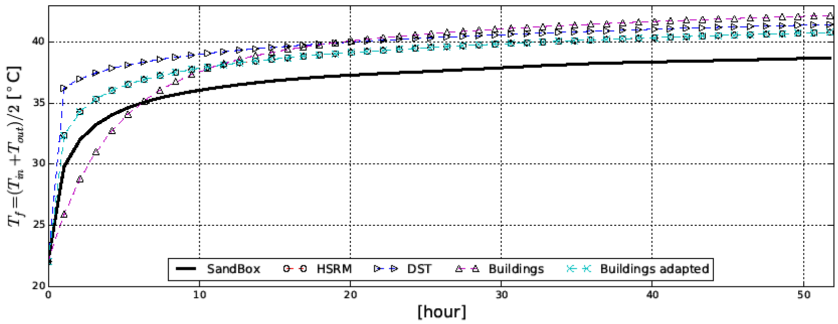

Figure 3: Comparison between the heat carrier fluid temperature from the sandbox experiment [5], the borehole model from the Buildings library, type 557a of TRNSYS (DST) and the new hybrid model (HSRM).

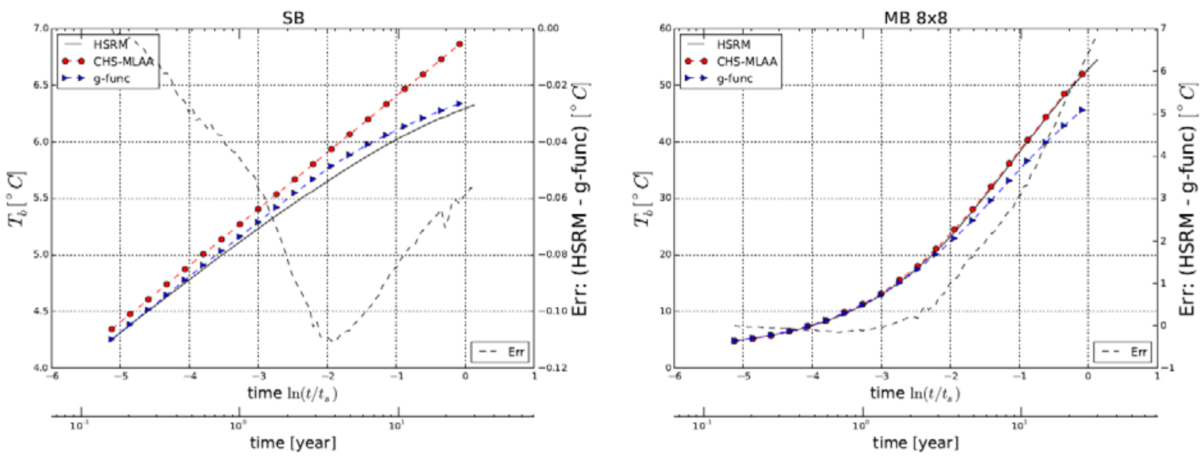

Figure 4: Average temperature step response of the borehole wall by the g-function g-func, the infinite cylindrical source and the new hybrid model HSRM (B/H=0.05, k=3.5 W/mK ((24.3 BTU.in/(hr.ft^2.F))*), a=1.62 mm^2/s (1.7x10^{-5} ft^2/s), H=110 m (328 ft).*

for (see Section 2.2). The end effect error is negligible compared to the large error (> 7 K after 25 years for this case, relative error of 17.5 %) introduced by the homogeneous heat source approximation. However, if the borefield is dissipative enough, the model shows very good results (e.g. for a line configuration of eight boreholes with the same parameter values, the error is lower than 0.1 K). For simulations with small yearly thermal ground imbalance (amount of injected heat ≈ amount of extracted heat), the configuration error is partly counteracted and it will not cause significant accuracy issues.

EXAMPLE CALCULATION

This section describes an example of a borefield subjected to a varying non-symmetric load with a time-step of 4 hours proposed by Bernier et al. [7]. The CPU time and the fluid temperature of the Buildings model and those of the HSRM model are compared for a simulation of one year in the case of a single borehole and the case of three boreholes in series (Fig. 5). The Buildings model is composed of the Buildings component *Boreholes.UTube*, an ideal heater and a pump.

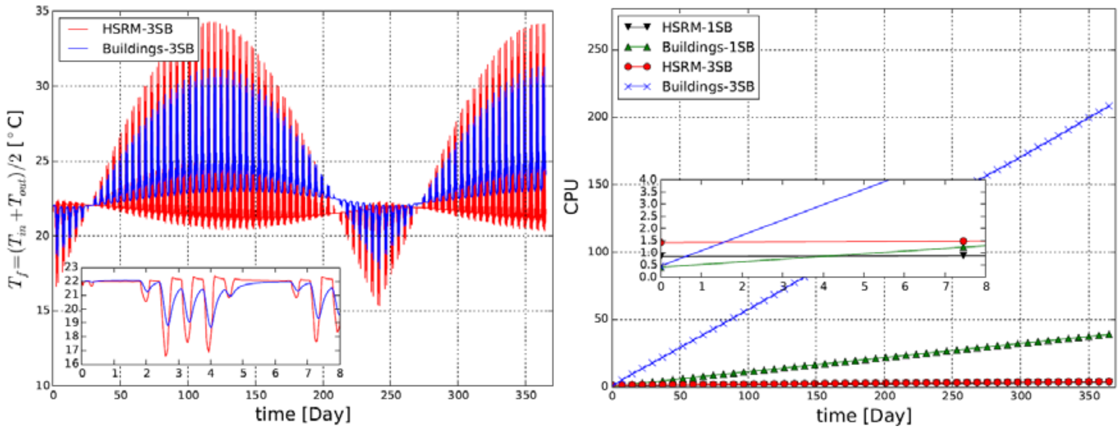

Figure 1: Left: heat carrier temperature for HSRM-3BH and Buildings-3BH. Right: CPU time comparison between the new model (HSRM) and the model from the Buildings library (Buildings) for a single borehole (1SB) and for three boreholes in series (3BH).

boreholes is taken into account by the SRM but not by the Buildings model.

As observed above, the Buildings model overestimates the time constant of the BHX which is also visible in Fig. 5 where the fluctuations of the HCF temperature of the Buildings model have a smaller amplitude than those of the HSRM model.

The analysis of the CPU times illustrates very clearly the difference between the models. In the case of a single borehole, the HSRM model is about twelve times faster than the Buildings model. The HSRM has a longer initialization time due to the calculation of the aggregation matrix, but it calculates the temperature response very fast. In the case of three boreholes in series, the HSRM is about 60 times faster. The initialization time is longer than for a single borehole because the superposition of the temperature field of the boreholes needs to be calculated. However, once the aggregation matrix is calculated, the calculation time is the same for any configuration. This is not the case for the Buildings model.

CONCLUSION

A new hybrid model for borefields with arbitrary configuration having both short-term (minutes) and long-term accuracy (decades) has been successfully developed and implemented in Modelica. The state-of-the-art of both short-term and long-term models has been combined into a hybrid model to calculate accurate step-responses (or so-called *g-functions*). The step-responses are superposed to get the fluid temperature response to an arbitrary load. Thanks to its aggregation method, the implemented model is about twelve times faster than the borehole model of the Buildings library for the case of a single borehole and about 60 times faster for the case of three boreholes in series. The long-term accuracy of the model decreases for compact borefield configuration. This can be solved by plugging a g-function in the model instead of calculating the temperature step response.

ACKNOWLEDGMENT

The authors acknowledge the financial support by IWT and WTCB in the frame of the IWT-VIS Traject SMART GEOTHERM focusing on integration of thermal energy storage and thermal inertia in geothermal concepts for smart heating and cooling of (medium) large buildings. Moreover, this study is part of the development work performed within IEA-ECB-Annex 60 on new generation computational tools for buildings and community energy systems based on the Modelica and Functional Mockup Interface standards.

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